1. Definite Integrals

1.a. ∫₄⁹ 3√x dx

Rewriting the integrand,

3√x = 3x^(1/2)

Finding the antiderivative,

∫ 3x^(1/2) dx = 3 \* (2/3) \* x^(3/2) = 2x^(3/2)

Evaluating the definite integral.

= [2x^(3/2)]₄⁹ = 2(9)^(3/2) - 2(4)^(3/2).

= 54 - 16 = 38.

Final Answer: 38.

1.b. ∫₁ᵉ ln(x) dx

Using integration by parts: ∫ u dv = uv - ∫ v du.

Let u = ln(x), dv = dx. Then, du = (1/x) dx and v = x.

∫ ln(x) dx = xln(x) - ∫ x \* (1/x) dx = xln(x) - x.

Evaluating the definite integral.

= [xln(x) - x]₁ᵉ = [eln(e) - e] - [1ln(1) - 1].

= [e - e] - [0 - 1] = 1.

Final Answer: 1.

1.c. ∫₀¹ cos⁻¹(x) dx

Finding the antiderivative of cos⁻¹(x).

The antiderivative of cos⁻¹(x) is xcos⁻¹(x) - √(1 - x²).

Evaluating the definite integral.

= [xcos⁻¹(x) - √(1 - x²)]₀¹.

= [1cos⁻¹(1) - √(1 - 1²)] - [0cos⁻¹(0) - √(1 - 0²)].

= [1(0) - 0] - [0(π/2) - 1].

= 0 - (-1) = 1.

Final Answer: 1.

1.d. ∫₋₁¹ πcos(πx/2) dx

Finding the antiderivative of πcos(πx/2).

The antiderivative is (2/π)sin(πx/2).

Evaluating the definite integral.

= [(2/π)sin(πx/2)]₋₁¹.

= [(2/π)sin(π/2)] - [(2/π)sin(-π/2)].

= [2/π] - [-2/π] = 4.

Final Answer: 4.

2. Indefinite Integrals

2.a. ∫ x²cos(x³) dx

Using substitution. Let u = x³, so du = 3x² dx.

Rewriting the integral: ∫ x²cos(x³) dx = (1/3)∫ cos(u) du.

Finding the antiderivative.

(1/3)∫ cos(u) du = (1/3)sin(u).

Substituting back u = x³.

= (1/3)sin(x³).

Final Answer: sin(x³)/3.

2.b. ∫ cos(3t)/(1 + sin(3t)) dt

Using substitution. Let u = 1 + sin(3t), so du = 3cos(3t) dt.

Rewriting the integral: ∫ cos(3t)/(1 + sin(3t)) dt = (1/3)∫ (1/u) du.

Finding the antiderivative.

(1/3)∫ (1/u) du = (1/3)ln|u|.

Substituting back u = 1 + sin(3t).

= (1/3)ln(1 + sin(3t)).

Final Answer: (1/3)log(1 + sin(3t)).